JCSS Joint Committee on Structural Safety

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BACKGROUND DOCUMENTATION

QUANTIFYING THE VALUE OF STRUCTURAL HEALTH INFORMATION FOR DECISION SUPPORT

FIRST EDITION

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PREFACE

The Guides for Operators, Scientists and Practicing Engineers on Quantifying the Value of Structural Health Information (SHI) for Decision Support have emerged from the scientific networking project COST Action TU1402 (www.cost-tu1402.eu) in the period from 2014 to 2019. The guides are the result of the TU1402 Working Group 5 on Standardisation in conjunction with the work of the Joint Committee on Structural Safety (JCSS – www.jcss-lc.org).

The Guide for Operators contains recommendations for the use of SHI value analyses by infrastructure owners, operators and authorities aiming at an enhanced infrastructure performance and utility management in terms of costs, life safety and sustainability.

The Guide for Scientists provides a consistent formulation of value of SHI decision scenarios encompassing probabilistic SHI system performance and cost models, probabilistic infrastructure performance and utility models and approaches for adapting infrastructure performance models with SHI. The Guide for Practicing Engineers aims to provide guidance in applying, implementing and using results of value of SHI analyses.

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1 INTRODUCTION

Information gathered by Structural Health Information (SHI) can substantially contribute to an enhanced performance and utility of civil structures and infrastructures. SHI may incorporate spatially local or global and temporal discrete or continuous information obtained by e.g. inspections, damage detection, load testing, structural health monitoring, non-destructive testing, big data, digital technologies and networks and industry 4.0.

Only when SHI are relevant for decisions influencing the performance and utility of infrastructures, SHI will lead to improving the sustainability, benefit generation and reduction of operational costs and risks throughout the life cycle of infrastructures. The relevance of SHI for the infrastructure performance and utility facilitates guidance of the design and innovation of SHI systems.

A systematic approach to quantify the relevance, i.e. the value, of SHI is provided by the Bayesian reliability, utility and decision analyses. Such analyses necessitate the modelling of decision scenarios involving (1) models of the infrastructure system and SHI system life cycle performance costs and consequences, (2) the definition of an objective function representing the decision maker's preferences and goals and (3) decision variables associated to the SHI systems and infrastructure system performance and utility management strategies.

This Guide for Scientists on Quantifying the Value of Structural Health Information for Decision Support provides a consistent and comprehensive formulation of the value of SHI quantification encompassing:

(1) Decision scenario modelling with a list of exemplary scenarios,
(2) The description of the decision theoretical foundation,
(3) The ranking of the SHI strategies on the basis of the value of information,
(4) The probabilistic modelling of SHI and a classification of SHI strategies and
(5) A list of approaches to update and adapt the structural performance with SHI

2 VALUE OF STRUCTURAL HEALTH INFORMATION

The quantification of the value of structural health information (SHI) takes basis in the Bayesian decision analyses (Raiffa and Schlaifer (1961)) in conjunction with utility theory (Von Neumann and Morgenstern (1947)) and its interpretation in the JCSS document on Risk Assessment in Engineering (Faber (2008)). The quantification of the value of SHI requires the definition of a decision scenario consisting of coupled infrastructure and a SHI systems models, an objective function, the SHI system decision variables and the temporal dimension of the decision analysis.

2.1 DECISION SCENARIO MODELLING AND ANALYSIS

The decision scenario for a value of SHI analysis consists of coupled infrastructure and SHI systems models, an objective function, the SHI system decision variables and the temporal dimension of the decision analysis. The decision scenario without the temporal dimension may be illustrated with a decision tree (Figure 1, left). Here, the SHI strategy i is described with the information type and its probabilistic outcomes, and the infrastructure performance with the system state models and the actions. The decision tree contains decision nodes to decide upon the utilisation of SHI, the SHI strategy to be utilised and the actions to be performed. The chance nodes stand for the probabilistic models associated to the SHI outcomes and the system states. The utility nodes encompass the decision attributes like benefits, costs and consequences associated to each of the decision tree branches.

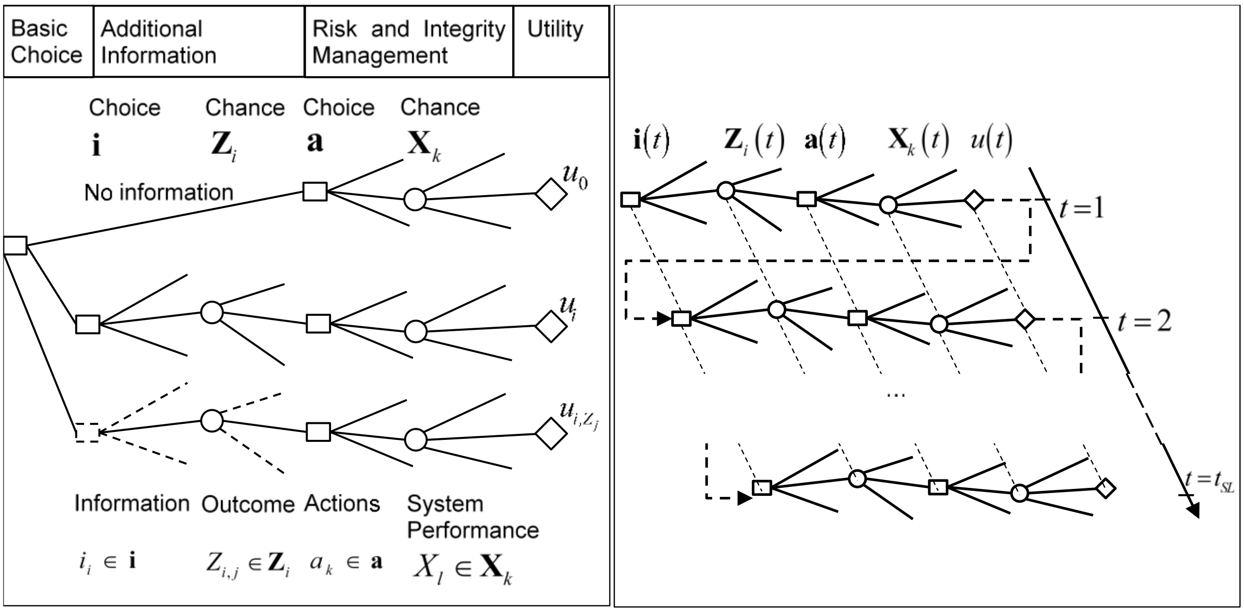


Figure 1: Decision tree for the assessment of the value of a SHI strategy containing decision nodes (rectangulars), chance nodes (circles) and utility nodes (diamonds) with predicted information (continuous lines) and obtained information (dashed lines for non-obtained information). Sequential pre-posterior decision analyses in the time domain (right).

When the decision tree is applied sequentially, it may contain the information gathered in previous time steps and (sub-)periods and common influencing variables. This is symbolized in Figure 1 (right) with the connections between the decision trees in different time steps and with the connections of the decision, chance and utility nodes, respectively.

The temporal formulation encompasses the dependencies of information, outcomes, actions, system states and the benefits between themselves and in succession.
Examples of the formulation of dependencies

- System state of fatigue: damage accumulation (explicit empirical formulation)
- FEM analysis of a system (explicit mechanical formulation)
- Results of successive inspections: by correlating the indica^^tion events (probabilistic formulation)

The temporal modelling of the decision analyses should be allocated and connected to the life cycle phases and the performance of the infrastructure system. The life cycle phases encompass the planning and design, the manufacturing, the construction, the operation and maintenance as well as decommissioning, see Figure 2. The life cycle phase models should be connected to the infrastructure system states namely the intact state, hazard state, constituent and system damage and failure states.

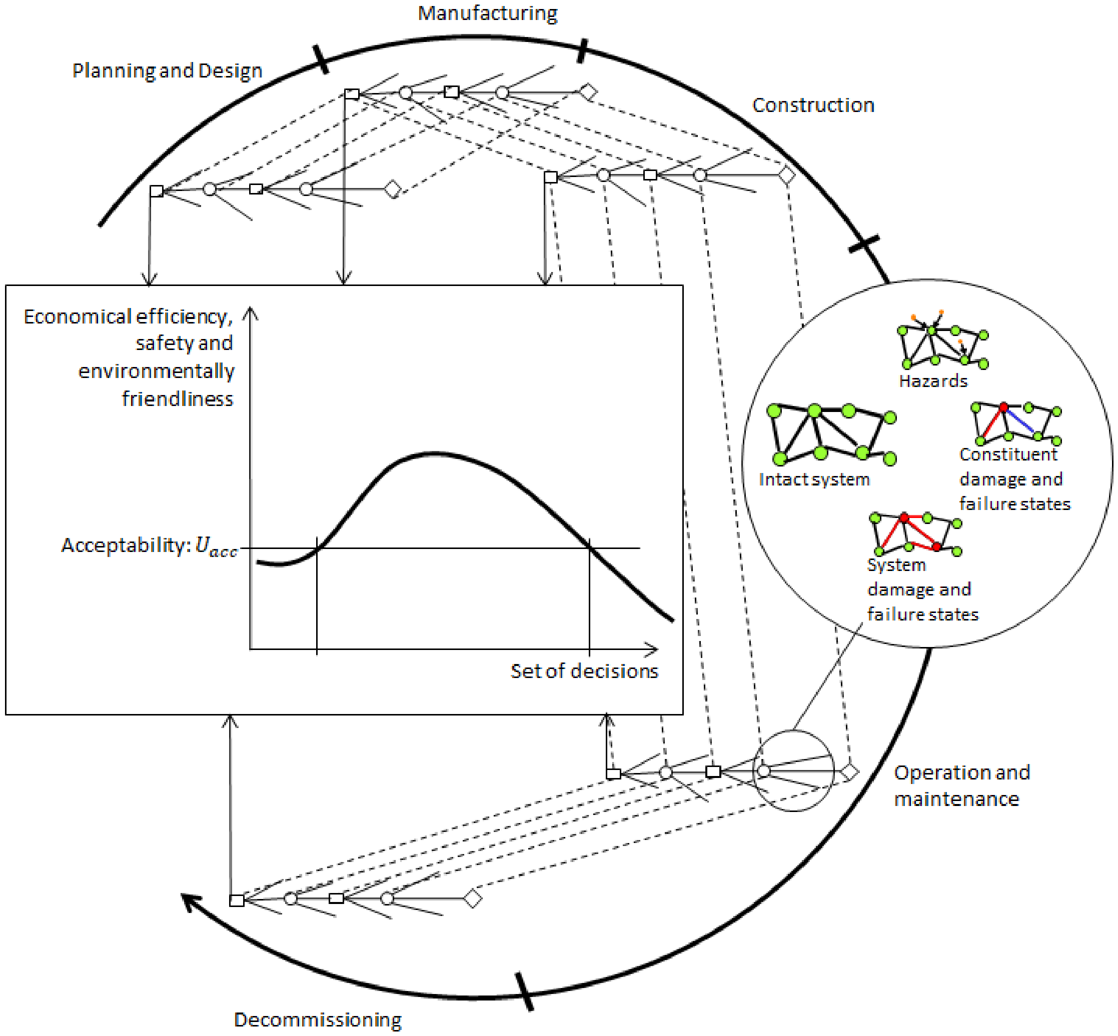


Figure 2: Illustration of a value of SHI analyses throughout the life cycle phases accounting for the intact, hazard, constituent and system damage and failure states (see also Faber, Qin et al. (2017)).

On the basis of the presented framework it becomes obvious that SHI may be of benefit throughout the life cycle of the structure and in any of the constituent and system states. Exemplary decision scenarios may be:

1. Code and standard calibration - decision support in the design phase

Code calibration for structure types may benefit from SHI when conducted systematically. The acquired information can be used to adapt the design basis in order to spare material and monetary resources while controlling safety, risk and reliability at the desired level. This could be e.g. achieved by reducing the model uncertainties in the design code equations with measuring the relevant magnitudes in the operational structure.

2. Structure prototype development and design by testing – design phase

The production of larger quantities of identical structures can benefit from optimisation processes supported by SHI. A prototype may be equipped with SHI systems in order to attain an optimized structural design before mass production. The SHI data may contain information to reduce uncertainties considered in the design model. Prediction of the response and performance of the prototype and mass produced structure may become more accurate with the use of SHI. The optimized design may thus lead to an increased life-cycle benefit of the structure.

3. Utility management in the operation phase of an infrastructure system

In the operation phase, SHI is often utilised for condition assessment, which may serve multiple purposes to plan an efficient utility, functionality, integrity and risk management. Typical purposes include:

a) Integrity management planning

The integrity management encompassing condition assessment, repair and maintenance actions should be planned and supported with tailored SHI system in the design phase and then continuously adapted to the obtained SHI.

b) Service life extension

For highly utilised structures, an extension of the service life is often desired. Here, SHI provide means and help for the condition assessment and condition prognosis which form the basis for the optimisation for ensuring the structural integrity beyond the service life.

c) Utilisation modification

The utilisation of structures may change throughout the service life, especially for infrastructures, examples are bridges which sustain a steadily increasing traffic load and e.g. re-powering of wind energy turbines. Here, data about the past utilisation of the structure, the actual condition and performance may provide benefit for the structural integrity management.

d) Functionality enhancement

The utility associated to the functionality of an infrastructure can be supported by the reduction of downtime and closures. SHI may support e.g. the integrity management planning enhancing the functionality can be obtained. This is of special importance for critical infrastructure systems which support vital societal functions. SHI can also be useful to mobilize resources for efficient response and recovery during and after breakdown of functionality and thereby enhancing the system's resilience.

e) Damage progression monitoring

In case that damage has been identified already, SHI systems may identify eventual trends (i.e. opening of cracks) to monitor eventual increased accelerated deterioration. From one or few monitored locations the damage progression at other locations maybe inferred.

f) Early damage warning for risk mitigation measures

SHI may indicate abnormal performance or possible damage of a structure and thus aid as indicators for remedial actions. In this application of SHI, the value of information would relate to the possibility of loss reduction by shutting down the function or reducing the loading of the structure, before human life, the structure and the environment are lost and/or damaged further. Embedded in a maintenance scheme, synergies with the structural integrity management and operation (see point (1)) can be realised.

4. SHI system development

SHI systems may be designed and optimized using value of information approaches closely associated to a decision scenario. The SHI system can then be specifically designed and the technology, number and placement of sensors and the measurement periods can be optimised.

2.2 SYSTEMS, UTILITY, CONSEQUENCE AND ACCEPTANCE CRITERIA MODELLING

The quantification of the value of SHI requires the definition of a decision scenario consisting of coupled - most commonly with a likelihood - infrastructure and a SHI systems models, an objective function, the SHI system decision variables and the temporal dimension of the decision analysis.

The infrastructure system is described with its system states, the associated utilities and physical measures influencing the system states namely actions like e.g. repair and strengthening. These performance models of the infrastructure system should account for the intact state, hazard state, constituent and system damage and failure states including their dependencies and probabilistic characteristics.

The SHI system is described with the information type and its probabilistic outcomes, which can serve as decision variables. The SHI influence to all relevant infrastructure system states, the associated utilities and actions needs to be explicitly modelled and all SHI system costs throughout the SHI system service life are to be accounted for.

All relevant utilities associated to the entire decision scenario including the system states and the physical measures (actions) and information should be modelled, quantified and discounted with an appropriate discount rate applicable to decision scenario, system boundaries and time horizon. The utilities should encompass generated benefits due to the infrastructure performance and consequences including (i) direct consequences due to constituent and system damage and (ii) indirect consequences, such as (a) loss of life, (b) economic losses, reduction in GDP, etc., (c) social losses and (d) environmental losses.

The system state modelling may build upon a risk analysis and it may be required that risk acceptance criteria have to be accounted for. In this case, standards and regulations applicable to the temporal and spatial scope of the decision analysis must be fulfilled. Present standards and regulations often specify occupational or working risks to individuals and/or risks to society. For societal risks, target probabilities can be derived by utilising the Life Quality index and the marginal live saving principles (see Faber (2008) and ISO 2394 (2015)). This facilitates that the societal preferences and capabilities can be accounted for. The target probabilities can then be compared with the quantified component and system failure probabilities. More details and models can be found in the JCSS document on Risk Assessment in Engineering (Faber (2008)) and the JCSS Probabilistic Model Code.

2.3 OBJECTIVE FUNCTIONS

The objective function models the decision maker's preferences aggregating the system performance, system management and utilities according to the decision scenario. The objective function addresses (1) the fundamental decision of considering additional and yet unknown information or not, (2) the identification of an optimal SHI strategy, (3) the identification of optimal actions given SHI and (4) the identification of optimal actions without SHI.

The fundamental decision of considering additional and yet unknown information or not can be based upon maximization of the value of information V. The maximisation of the value of information is calculated maximising the expected value of the utilities with and without SHI, $U_1\left(i_i^*,\mathbf{a}^{*,i}\right)$ and $U_0\left(\mathbf{a}^{*,0}\right)$, respectively, which is performed with the identification of the optimal SHI strategy i_i^*, the SHI outcomes dependent optimal subset of actions $\mathbf{a}^{*,i}\subset\mathbf{a}$ and the optimal action without SHI, $a_k{}^0\in\mathbf{a}$.

$$V\left(i_i^*,\mathbf{a}^{*,i},a_k{}^{*,0}\right)=U_1\left(i_i^*,\mathbf{a}^{*,i}\right)-U_0\left(a_k{}^{*,0}\right)\text{ with}$$
$$\left\{i_i^*,\mathbf{a}^{*,i},a_k{}^{*,0}\right\}=\underset{i_i\in\mathbf{i},\mathbf{a}^i\subset\mathbf{a}}{\arg\max}\,U_1\left(i_i,\mathbf{a}^i\right)-\underset{a_k{}^0\in\mathbf{a}}{\arg\max}\,U_0\left(a_k{}^0\right)$$
$$\text{s.t. }U_{acc}\le U_1\left(i_i^*,\mathbf{a}^{*,i}\right)\text{ and }U_{acc}\le U_0\left(a_k{}^{*,0}\right)\tag{1}$$

The Value of Information can be normalized in relation to the prior life cycle benefits U_0 resulting in the relative Value of Information, i.e.:

$$\bar{V}\left(i_i^*,\mathbf{a}^{*,i},a_k{}^{*,0}\right)=\frac{U_1\left(i_i^*,\mathbf{a}^{*,i}\right)-U_0\left(a_k{}^{*,0}\right)}{U_0\left(a_k{}^{*,0}\right)}\text{ s.t. }U_{acc}\le U_1\left(i_i^*,\mathbf{a}^{*,i}\right)\text{ and }U_{acc}\le U_0\left(a_k{}^{*,0}\right)\tag{2}$$

The decisions about (2) the identification of an optimal SHI strategy, (3) the identification of optimal actions given SHI and (4) the identification of optimal actions without SHI are modelled with a pre-posterior and a prior or a posterior decision analysis.

The expected value of the utility with no SHI involving the decision about an optimal action, $U_0\left(a_k{}^{*,0}\right)$, is calculated with the maximisation of the action and chance of the system's life cycle performance $X_l\in\mathbf{X}_k$ dependent utility $u_0\left(a_k,X_l\right)$:

$$U_0\left(a_k{}^{*,0}\right)=\max E_{X_l}\left[u_0\left(a_k{}^{*,0},X_l\right)\right]\text{ with }a_k{}^{*,0}=\underset{a_k\in\mathbf{a}}{\arg\max}\left(E_{X_l}\left[u_0\left(a_k,X_l\right)\right]\right)$$

$$\text{s.t. } U_{acc} \leq U_0\left(a_k^{*,0}\right) \tag{3}$$

For the identification of an optimal SHI strategy and the optimal actions given SHI the expected value of the utility $U_1\left(i_i^*, \mathbf{a}^{*,i}\right)$ can be calculated in the extensive form. Here, the decision tree is analysed from the right hand side of the decision tree, i.e. from the life cycle performance to the initial starting point: the choice of the information. The expectation over the posterior life cycle performance is calculated by Bayesian updating (operator E''_{X_l}) and the dependency on the life cycle performance is then marginalized with the expectation in regard to the chances of the information (Z_j) outcomes E_{Z_j}:

$$U_1\left(i_i^*, \mathbf{a}^{*,i}\right) = E_{Z_j}\left[E''_{X_l}\left[u_i\left(i_i^*, Z_j, \mathbf{a}^{*,i}, X_l\right)\right]\right] \text{ with}$$

$$\left\{i_i^*, \mathbf{a}^{*,i}\right\} = \underset{i_i \in \mathbf{i}, \mathbf{a}^i \subset \mathbf{a}}{\arg\max} E_{Z_j}\left[\underset{a_k \in \mathbf{a}}{\arg\max} E''_{X_l}\left[u_i\left(i_i, Z_j, a_k, X_l\right)\right]\right]$$

$$\text{s.t. } U_{acc} \leq U_1\left(i_i^*, \mathbf{a}^{*,i}\right) \tag{4}$$

In the normal form, the decision tree is defined with the decision rules $d_m\left(Z_{i,j}, a_k\right) \in \mathbf{d}(\mathbf{Z}_i, \mathbf{a})$ linking the actions with the random outcomes of the information acquirement strategy i. The decision tree is analysed starting from the left hand side calculating the expected value in regard to the information outcome for each given life cycle performance with the operator $E_{Z_j|X_l}$. Consecutively, the expectation in regard to the life cycle performance is calculated, i.e. the dependent benefits are marginalized:

$$U_1\left(i_i^*, \mathbf{d}^*\right) = E_{X_l}\left[E_{Z_j|X_l}\left[u_i\left(i_i^*, Z_j, \mathbf{d}^*, X_l\right)\right]\right] \text{ with}$$

$$\left\{i_i^*, \mathbf{d}^*\right\} = \underset{i_i \in \mathbf{i}, \mathbf{d}^i \subset \mathbf{d}}{\arg\max} E_{X_l}\left[E_{Z_j|X_l}\left[u_i\left(i_i, Z_j, \mathbf{d}^i, X_l\right)\right]\right]$$

$$\text{s.t. } U_{acc} \leq U_1\left(i_i^*, \mathbf{d}^*\right) \tag{5}$$

For the case that the SHI has already been acquired, the posterior value of SHI $V \mid Z_j$ can be assessed retrospectively as the difference of the expected value of the utilities with and without the SHI Z_j, $U_2\left(a_k^{*,i}\right)$ and $U_0\left(a_k^{*,0}\right)$. A posterior value of SHI analysis addresses the identification of optimal actions given SHI and the identification of optimal actions without SHI. The value of information is maximised by identifying the optimal actions with and without the SHI, $a_k^{*,i}$ and $a_k^{*,0}$ respectively.

$$V \mid Z_j = U_2\left(a_k^{*,i}\right) - U_0\left(a_k^{*,0}\right) \text{ with } a_k^{*,i} = \underset{a_k^i \in \mathbf{a}}{\arg\max} U_2\left(a_k^i\right) - U_0\left(a_k^{*,0}\right) \tag{6}$$

$$\text{s.t. } U_{acc} \leq U_2\left(a_k^{*,i}\right) \text{ and } U_{acc} \leq U_0\left(a_k^{*,0}\right) \tag{7}$$

The expected value of the utility U_2 involving the decision about the identification of optimal actions without SHI with SHI is calculated with the posterior expectation and the optimal action $a_k^{*,i}$:

$$U_2\left(a_k^{*,i}\right)=E''_{X_l}\left[u_{i,Z_j}\left(a_k^{*,i},X_l\right)\right] \text{ with } a_k^{*,i}=\arg\max_{a_k\in\mathbf{a}} E''_{X_l}\left[u_{i|Z_j}\left(a_k,X_l\right)\right] \text{ s.t. } U_{acc}\leq U_2\left(a_k^{*,i}\right) \quad (8)$$

2.4 DOCUMENTATION AND RANKING OF THE VALUE OF STRUCTURAL HEALTH INFORMATION

The value of structural health information documentation should encompass a detailed listing of the considered decision scenario, its boundaries, the objective function, the decision alternatives and the expected value of the utilities for each of the decision alternatives.
The full set of decision alternatives referring (1) actions, (2) actions given an SHI outcome (so called decision rules), (3) the ranking of SHI strategies and (4) the ranking of the binary decision alternatives of utilising SHI or not is to be documented.

3 STRUCTURAL HEALTH INFORMATION

Decisions contributing to an efficient management of infrastructures rely on information conditions. According to Nielsen, Glavind et al. (2018), it can be distinguished between:

1) The information is relevant and precise.
2) The information is relevant but imprecise.
3) The information is irrelevant.
4) The information is relevant but incorrect.
5) The flow of information is disrupted or delayed.

From this context, it follows that structural health information is information with relevance for the decisions influencing the infrastructure performance and utility. SHI are thus required to be relevant by the temporal period and for the structural functionality, damage and failure events, their consequences and the actions. A SHI model encompasses the information type and content, the probabilistic properties and the costs and consequences. The boundaries of the probabilistic SHI models should be well documented to account for information conditions (above).

3.1 SHI CLASSIFICATION

SHI are classified in regard to (1) the relation to the structural system state models by the SHI type (direct measurement of structural system model parameter or indirect event probability information), (2) structural system space (e.g. constituent, subsystem, system) and (3) temporal period within a structural system life cycle period (e.g. discrete, periodically, continuous).Table 1 lists examples of SHI.

Table 1: SHI type examples

SHI	Type	Direct / indirect	Spatial characteristic	Temporal characteristic
Damage detection with a distributed measurement systems based on the analysis of static and dynamic behaviour	Damage event	Indirect (indication of damage event)	System or subsystem level	Continuous or periods of measurements
Inspection	Damage event	Indirect (indication of	Constituent	Discrete

		damage)		
Load testing	Survival event	Indirect (indication of survival or damage or failure)	Constituent, subsystem or system level	Discrete
Monitoring of constituent loading	Load measurement	Direct	Constituent	Continuous or periods of measurements
Non-destructive testing	Resistance measurement	Direct	Constituent	Discrete

3.2 PROBABILISTIC CHARACTERISTICS OF SHI

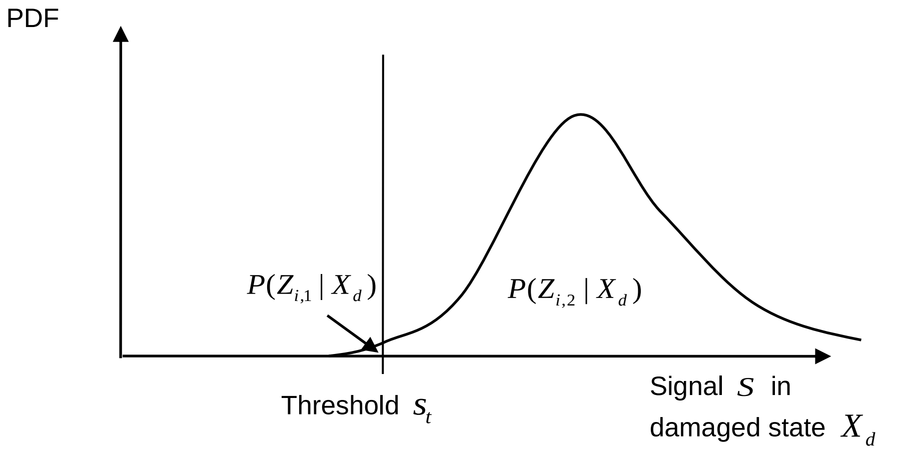
The SHI information have probabilistic characteristics originating from (1) the measurement process, (2) the SHI installation and operation and (3) the data analysis. Probabilistic characteristics of the measurement process are caused by the conversion of e.g. electrical or optical signals to structural properties and the process inherent uncertainties relating to the sensor precision, conversion and amplification unit.

During the SHI installation and operation, the probabilistic model should account for (a) the sensor and SHI system installation, (b) the operational conditions, which are not covered by data normalisation and (c) human errors.

The probabilistic models referring to the data analysis should account for (a) the statistical uncertainties due to a limited amount of data and limited measurement periods in relation to the temporal boundaries of the decision scenario, (b) the limited precision of data analysis and data normalisation algorithms and (c) human errors. For consecutive or multiple SHI, the dependency characteristics must be explicitly modelled.

The uncertainties related to the measurement process (see (1)) and the SHI installation and operation (see (2)) can be reduced by calibration of the measurement system and by measuring under specified conditions. The data analysis uncertainties require in most cases explicit probabilistic modelling.

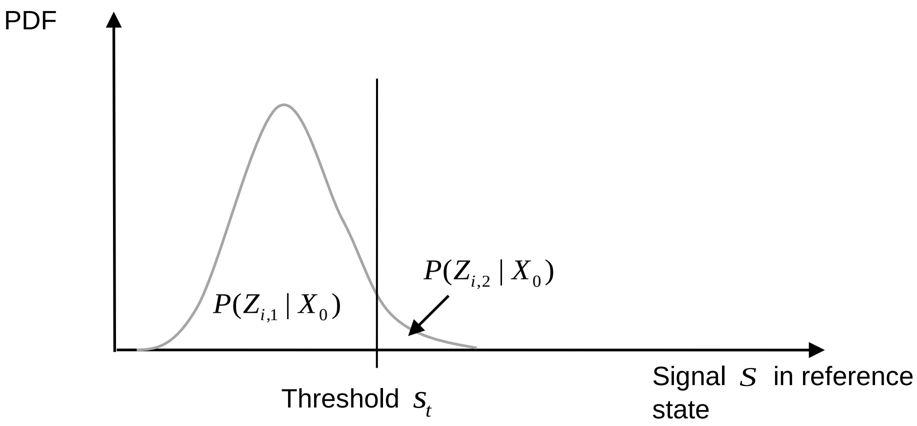
The uncertainties originating from the measurement process, the SHI installation and operation and the data analysis may be modelled based on detection theory, see e.g. Kay (1993) and Gandossi and Annis (2010). The signal distribution of a SHI system may be obtained for the reference state ($X_{l=0}$) and the damaged states with damage $X_{l=1...d}$ (Figure 3 and Figure 4). With the introduction of a threshold s_t , it can be distinguished between a reference state (e.g. safe) indication $Z_{i,1}$ and damage indication $Z_{i,2}$ for the SHI strategy i.



$$P(Z_{i,1} \mid X_d) = \int_{-\infty}^{s_t} f_S\left(s \mid X_d\right) ds \quad (9)$$

$$P(Z_{i,2} \mid X_d) = \int_{s_t}^{\infty} f_S\left(s \mid X_d\right) ds \quad (10)$$

Figure 3: Analytical formulation of the SHI system test outcomes given damage



$$P(Z_{i,1} \mid X_0) = \int_{-\infty}^{s_t} f_S\left(s \mid X_0\right) ds \quad (11)$$

$$P(Z_{i,2} \mid X_0) = \int_{s_t}^{\infty} f_S\left(s \mid X_0\right) ds \quad (12)$$

Figure 4: Analytical formulation of the SHI system test outcomes given no damage

The integration of the signal distributions for all damage states (accumulated in the vector $\mathbf{X}_d$) leads to the probability of damage indication curve $P\left(Z_{i,2}\left(\mathbf{X}_d\right)\right)$, see Figure 5.

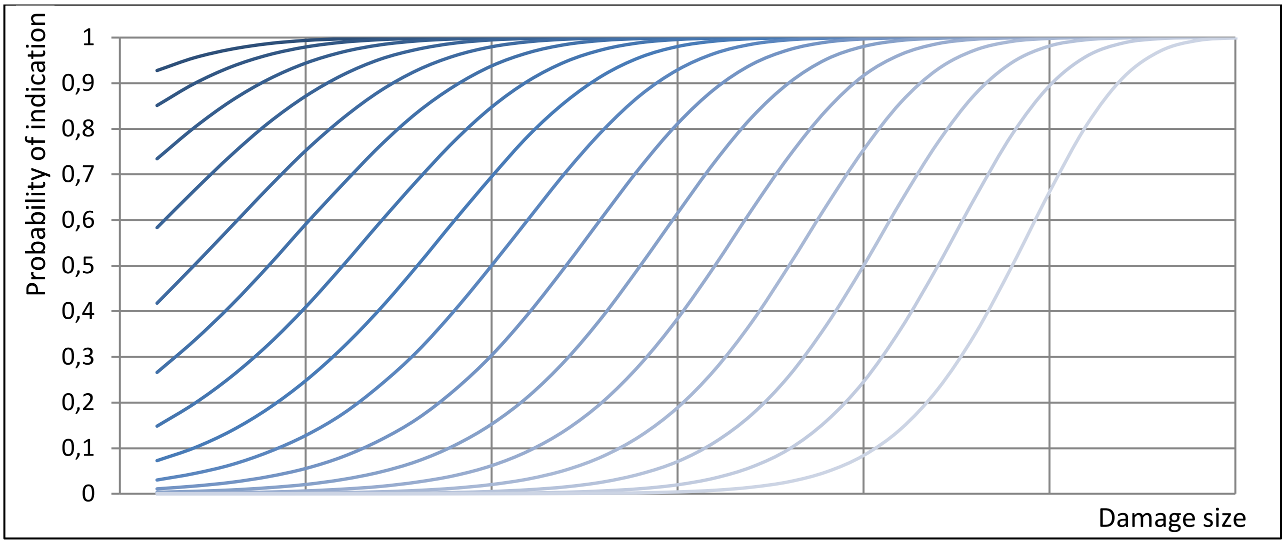


Figure 5: Probability of indication curves in the damaged states for different SHI strategies i dependent on the threshold (low threshold: dark blue; high threshold: light blue).

The probability of a safe indication given a damaged state can be calculated with $P\left(Z_{i,1}\left(\mathbf{X}_d\right)\right)=1-P\left(Z_{i,2}\left(\mathbf{X}_d\right)\right)$. The marginal probabilities of a safe and damage indication curves, which account for the false indications (safe indication but damage state $Z_{i,1}\left(\mathbf{X}_d\right)$ and damage indication but undamaged state $Z_{i,2}\left(X_0\right)$) are calculated with additional consideration of the reference state, i.e.:

$$P\left(Z_{i,2}\left(X_0,\mathbf{X}_d\right)\right) \text{ and } P\left(Z_{i,1}\left(X_0,\mathbf{X}_d\right)\right)=1-P\left(Z_{i,2}\left(X_0,\mathbf{X}_d\right)\right) \tag{13}$$

The formulation above can be extended to account for system reference ($X_{S,0}$) and damage states $\mathbf{X}_{S,d}$:

$$P\left(Z_{i,2,S}\left(X_{S,0},\mathbf{X}_{S,d}\right)\right) \text{ and } P\left(Z_{i,1,S}\left(X_{S,0},\mathbf{X}_{S,d}\right)\right)=1-P\left(Z_{i,2}\left(X_{S,0},\mathbf{X}_{S,d}\right)\right) \tag{14}$$

The marginal probabilities of a system damage indication are calculated by accounting for all system damage states $\mathbf{X}_{S,d}$ and the system reference state $X_{S,0}$ the resulting n_c-variate signal distribution $f_{n_c,S}$:

$$P(Z_{i,1,S} \mid X_{S,0},\mathbf{X}_{S,d}) = \int_{-\infty}^{s_t} f_{n_c,S}\left(s \mid X_{S,0},\mathbf{X}_{S,d}\right)ds \text{ and}$$

$$P(Z_{i,2,S} \mid X_{S,0},\mathbf{X}_{S,d}) = \int_{s_t}^{-\infty} f_{n_c,S}\left(s \mid X_{S,0},\mathbf{X}_{S,d}\right)ds \tag{15}$$

The high computational demands for the pre-calculation of the multivariate probability of indication curve can be overcome by exploiting specific characteristics of SHI algorithms facilitating a direct calculation of the probability of indication for any system state (Thöns, Döhler et al. (2018)).

3.3 SHI CONSEQUENCES AND COSTS

The cost and consequences of SHI system investment, installation, operation, renewal and decommissioning require explicit modelling. The service life of the constituent of the SHI and the according replacement intervals within the service life of the infrastructure system should be accounted for. Special attention and explicitness is required for modelling the consequences of SHI false indications.

4 STRUCTURAL PERFORMANCE ADAPTATION AND STRUCTURAL HEALTH INFORMATION MODELLING

The structural performance adaptation necessitates a consistent modelling of structural health information accounting for their type and probabilistic characteristics in conjunction with the structural system performance models on the basis of Bayesian probability, reliability and decision theory.

4.1 DAMAGE DETECTION INFORMATION MODELLING

Damage detection information may be modelled with an indication event such as provided with a damage detection system or an inspection of a system constituent. The modelling of the event requires a limit state function which accounts for the probabilistic information characteristics and is connected to the structural performance states X_l.

The pre-posterior probability (for a value of SHI analysis, i.e. a pre-posterior decision analysis) of the constituent state $X_{1,c}$ with damage detection information for constituents (e.g. inspection strategy i providing a vector of indications $\mathbf{Z}_i$) can be calculated by utilising the definition of dependent probabilities.

$$P\left(X_{1,c} \mid \mathbf{Z}_i\right)P\left(\mathbf{Z}_i\right)=P\left(\mathbf{Z}_i \mid X_{1,c}\right)P\left(X_{1,c}\right)=P\left(\mathbf{Z}_i \cap X_{1,c}\right) \tag{16}$$

Equ. (16) can be extended for n consecutive damage detection indications $\left(t_{i,1}\ldots t_{i,n}\right)$ throughout the service life and reformulated as a structural reliability problem with the joint distribution of the indications and constituent states random variables $f_{\mathbf{X}_{X_{1,c}(t)},\mathbf{X}_{\mathbf{Z}_i(t_1)},\ldots,\mathbf{X}_{\mathbf{Z}_i(t_n)}}\left(\mathbf{x}_{X_{1,c}(t)},\mathbf{x}_{\mathbf{Z}_i\left(t_{i,1}\right)},\ldots,\mathbf{x}_{\mathbf{Z}_i\left(t_{i,n}\right)}\right)$ integrated over the limit state function defined domains $\Omega_{X_{1,c}(t)} \cap \Omega_{\mathbf{Z}_i\left(t_{i,1}\right)} \cap \ldots \cap \Omega_{\mathbf{Z}_i\left(t_{i,n}\right)}$:

$$P\left(X_{1,c}\left(t\right)\cap\mathbf{Z}_i\left(t_{i,1}\right)\cap\ldots\cap\mathbf{Z}_i\left(t_{i,n}\right)\right)$$
$$=\int\limits_{\Omega_{X_{1,c}(t)}\cap\,\Omega_{\mathbf{Z}_i\left(t_{i,1}\right)}\cap\ldots\cap\Omega_{\mathbf{Z}_i\left(t_{i,n}\right)}} f_{\mathbf{X}_{X_{1,c}(t)},\mathbf{X}_{\mathbf{Z}_i(t_1)},\ldots,\mathbf{X}_{\mathbf{Z}_i(t_n)}}\left(\mathbf{x}_{X_{1,c}(t)},\mathbf{x}_{\mathbf{Z}_i\left(t_{i,1}\right)},\ldots,\mathbf{x}_{\mathbf{Z}_i\left(t_{i,n}\right)}\right)d\mathbf{x}_{X_{1,c}(t)}d\mathbf{x}_{\mathbf{Z}_i\left(t_{i,1}\right)}\ldots d\mathbf{x}_{\mathbf{Z}_i\left(t_{i,n}\right)} \tag{17}$$

Once a damage detection indication has been obtained, the posterior probability (for a posterior value of SHI analysis, i.e. a posterior decision analysis) of the constituent state $X_{1,c}$ with the constituent damage detection information Z_j can be calculated by utilising Bayesian updating:

$$P\left(X_{1,c} \mid Z_{i,j}\right)=\frac{P\left(Z_{i,j} \mid X_{1,c}\right)P\left(X_{1,c}\right)}{P\left(Z_{i,j}\right)}=\frac{P\left(Z_{i,j} \cap X_{1,c}\right)}{P\left(Z_{i,j}\right)} \tag{18}$$

For consecutive or multiple indications $\left(Z_{1\ldots j_n}\left(t_{i,1}\right)\ldots Z_{1\ldots j_n}\left(t_{i,n}\right)\right)$, the posterior probability of the constituent state $X_{1,c}$ can be calculated with extending Equ. (17) and (18), see Equ. (19).

$$P\left(X_{1,c}(t) \mid Z_{i,j}(t_{i,1}) \ldots Z_{i,j}(t_{i,n})\right) = \frac{P\left(X_{1,c}(t) \cap Z_{i,1\ldots j_n}(t_{i,1}) \cap \ldots \cap Z_{i,1\ldots j_n}(t_{i,n})\right)}{P\left(Z_{i,1\ldots j_n}(t_{i,1}) \cap \ldots \cap Z_{i,1\ldots j_n}(t_{i,n})\right)} \quad (19)$$

The dependencies of consecutive or multiple indications and/or of the individual parameters for describing the indication events need to be explicitly accounted for.

The probability of the constituent state $P\left(X_{1,c}\right)$ may be written as a structural reliability problem with where the domain Ω_{X_1} is defined with the limit state function:

$$P\left(X_{1,c}\right) = \int_{\Omega_{X_{1,c}}} f_{\mathbf{X}_{X_{1,c}}}\left(\mathbf{x}_{X_{1,c}}\right) d\mathbf{x}_{X_{1,c}} \quad (20)$$

The constituent limit state function $g_{X_{1,c}}\left(\mathbf{X}_{X_{1,c}}\right)$ may be exemplarily formulated with the damage dependent resistance $R_c(D_c)$ and loading S_c, which are subjected to the resistance and loading model uncertainties M_{R_c} and M_{S_c}, respectively:

$$X_{1,c}: \quad g_{X_{1,c}}\left(\mathbf{X}_{X_{1,c}}\right) = M_{R_c} R_c(D_c) - M_{S_c} S_c \leq 0 \quad (21)$$

The limit state function for the constituent no indication event Z_j is formulated following Hong (1997) with a damage size dependent probability of indication curve $P\left(Z_j(d=0,\mathbf{d})\right)$, the damage distribution D_c and a uniformly distributed random variable U:

$$Z_j: \quad g_{Z_{i,j}}\left(\mathbf{X}_{Z_{i,j}}\right) = P\left(Z_{i,j}(D_c)\right) - U \leq 0 \quad (22)$$

For damage detection information on structural system level, the probability of the system state $X_{1,S}$ can be calculated with the joint distribution of the system state random variables $f_{\mathbf{X}_{X_{1,S}}}\left(\mathbf{x}_{X_{1,S}}\right)$:

$$P\left(X_{1,S}\right) = \int_{\Omega_{X_{1,S}}} f_{\mathbf{X}_{X_{1,S}}}\left(\mathbf{x}_{X_{1,S}}\right) d\mathbf{x}_{X_{1,S}} \quad (23)$$

For the case that the system model can be represented with logical system modelling accounting the redundancy, i.e. statically indeterminacy, of the system, the probability of system failure can be calculated. The structural system is decomposed into n_{j_c} parallel and n_{i_c} series systems (Equ. (24)). The system limit state function can also be formulated considering the redundancy and the brittle or ductile component behaviour with Daniels system modelling.

$$P\left(X_{1,S}\right) = P\left(\bigcap_{j_c}^{n_{j_c}} \bigcup_{i_c}^{n_{i_c}} \left\{ g_{X_{1,i_c,j_c}}\left(\mathbf{X}_{X_{1,i_c,j_c}}\right) = M_{R_{i_c,j_c}} R_{i_c,j_c}\left(D_{i_c,j_c}\right) - M_{S_{i_c,j_c}} S_{i_c,j_c} \leq 0 \right\}\right) \quad (24)$$

The limit state function for the constituent no indication event Z_j is readily formulated with a damage size per constituent dependent probability of indication curve $P\left(Z_{1,S}\left(\mathbf{d}_{S,0},\mathbf{d}_S\right)\right)$ and the vector of constituent damage distributions

$\mathbf{D} = \left[D_{i_c, j_c} \ldots D_{n_{i_c}, n_{j_c}} \right]^T$:

$$Z_{1,S}: \quad g_{Z_{S,i,1}}\left(\mathbf{X}_{Z_{S,i,1}}\right) = P\left(Z_{S,i,1}\left(\mathbf{D}\right)\right) - U \leq 0 \qquad (25)$$

4.2 LOAD TESTING

The adaptation of the system state probabilities with survival information follows the principles as outlined in the previous Section with Equ. (16) to (19) with the constituent and system limit state functions Equ. (21), (23) and (24).
The load testing constituent survival event $Z_{1,c}$ can be modelled building upon the limit state function in Equ. (21) with consideration of the testing load $S_{c,T}$ and the associated model uncertainty $M_{S_{c,T}}$:

$$Z_{1,c}: \quad g_{Z_{1,c}}\left(\mathbf{X}_{Z_{1,c}}\right) = M_{R_c} R_c\left(D_c\right) - M_{S_{c,T}} S_{c,T} > 0 \qquad (26)$$

In specific cases upon justification, the load testing model uncertainty may be neglected and the testing may very controlled having negligible probabilistic characteristics:

$$Z_{1,c}: \quad g_{Z_{1,c}}\left(\mathbf{X}_{Z_{1,c}}\right) = M_{R_c} R_c\left(D_c\right) - s_{c,T} > 0 \qquad (27)$$

The load testing system survival event can be formulated with the proof loading constituent loadings and model uncertainties, $S_{i_c, j_c, T}$ and $M_{S_{i_c, j_c, T}}$, respectively:

$$P\left(Z_{S,i,1}\right) = P\left(\bigcap_{j_c}^{n_{j_c}} \bigcup_{i_c}^{n_{i_c}} \left\{ g_{Z_{1,i_c,j_c}}\left(\mathbf{X}_{Z_{S,1,i_c,j_c}}\right) = M_{R_{i_c,j_c}} R_{i_c,j_c}\left(D_{i_c,j_c}\right) - M_{S_{i_c,j_c,T}} S_{i_c,j_c,T} \leq 0 \right\}\right) \qquad (28)$$

In order to assess the feasibility of the proof load test, the expected gain due to improved information about structural resistance and risks associated with the test (i.e. risk of permanent damage or partial/ full collapse of a structure during the test) needs to be optimised. The past performance of the structure, i.e. that the structure has survived a period of time, should also be considered (see e.g. Faber, Val et al. (2000)).

4.3 MONITORING INFORMATION MODELLING

SHI may be modelled by taking basis in characteristics of model uncertainties. Model uncertainties apply to any parameter of the constituent and system model states and account for model inherent simplifications and assumption. Model uncertainties are determined by means of measurements (see e.g. JCSS (2006), Part 3.09). The process of determining the model uncertainties implies that a built structure constitutes a realization of the model uncertainty. Then, measuring on a structure provides information about the model uncertainty realization subjected to the probabilistic measurement information characteristics.
In the context of a pre-posterior decision analysis, the yet unknown monitoring information may be modelled with threshold-truncated distributions of the model uncertainty and the monitoring uncertainty. For the case of two thresholds $t_{M_{S,c},1}$ and $t_{M_{S,c},2}$, the probability of indications may be calculated by the integration of the truncated probability density. The thresholds are subjected to calibration (e.g. to target probabilities of component failure) and/or the optimisation. The indication event $Z_{i,1,c}$

may describe the situation of low model uncertainty realisations and consequently of a low constituent loading. The $Z_{i,2,c}$ indication describes a constituent, which performs around the expected value of the loading model uncertainty. The event $Z_{i,3,c}$ may describe the situation of a higher than expected realisation of the loading model uncertainty.

$$\begin{aligned} Z_{1,c}: \quad & P\left(Z_{i,1,c}\right)=\int_{-\infty}^{t_{M_{S,c},1}} f_{M_{S,c}}\left(m_{S,c}\right)dm_{S,c} \\ Z_{2,c}: \quad & P\left(Z_{i,2,c}\right)=\int_{t_{M_{S,c},1}}^{t_{M_{S,c},2}} f_{M_{S,c}}\left(m_{S,c}\right)dm_{S,c} \\ Z_{3,c}: \quad & P\left(Z_{i,3,c}\right)=\int_{t_{M_{S,c},2}}^{\infty} f_{M_{S,c}}\left(m_{S,c}\right)dm_{S,c} \end{aligned} \tag{29}$$

The probability of the constituent state $X_{1,c}$ may be calculated with a limit state function and the threshold-truncated random variable $M_{S,c}\left[t_{M_{S,c},1};t_{M_{S,c},2}\right]$ and the monitoring uncertainty $M_{U,c}$ (Equ. (30)).

$$g_{X_{1,c},Z_{2,c}}\left(\mathbf{X}_{X_{1,c}},Z_{i,2,c}\right)=M_{R_c}R_c\left(D_c\right)-M_{S,c}\left[t_{M_{S,c},1};t_{M_{S,c},2}\right]M_{U,c}S_c\leq 0 \tag{30}$$

Already obtained (posterior) measurement information are modelled with the realization $m_{S,c}$ and the monitoring uncertainty:

$$g_{X_{1,c},Z_{2,c}}\left(\mathbf{X}_{X_{1,c}},Z_{i,2,c}\right)=M_{R_c}R_c\left(D_c\right)-m_{S,c}M_{U,c}S_c\leq 0 \tag{31}$$

The dependencies for consecutive measurements may be modelled with the correlation of the monitoring uncertainty $M_{U,c}\left(t_{i,1}\right)...M_{U,c}\left(t_{i,n}\right)$ according to the dependency characteristics of the probabilistic measurement process, the SHI installation and operation and the data analysis models (see Section 0). For example, the installation and calibration uncertainties may be assumed fully correlated for an installed and calibrated monitoring system, whereas dependencies originating from the measurement process, the operation and the data analysis may be subjected to specific degrees of dependencies and randomness.

4.4 NON-DESTRUCTIVE OR DESTRUCTIVE TESTING

Non-destructive or destructive testing information can be modelled as information about the resistance model uncertainty in analogy to monitoring information for loading model uncertainties. The indication event $Z_{i,2,c}$ may calculated by integration of the resistance model uncertainty between two (defined, calculated and/or optimised) thresholds $t_{M_{R,c},1}$ and $t_{M_{R,c},2}$:

$$Z_{2,c}: \quad P\left(Z_{i,2,c}\right)=\int_{t_{M_{S,c},1}}^{t_{M_{R,c},2}} f_{M_{R,c}}\left(m_{R,c}\right)dm_{R,c} \tag{32}$$

The pre-posterior probability of the constituent state $X_{1,c}$ may be calculated with a limit state function and the threshold-truncated random variable $M_{R,c}\left[t_{M_{R,c},1};t_{M_{R,c},2}\right]$ and the monitoring uncertainty $M_{U,c}$:

$$g_{X_{1,c},Z_{2,c}}\left(\mathbf{X}_{X_{1,c}},Z_{i,2,c}\right)=M_{R,c}\left[t_{M_{R,c},1};t_{M_{R,c},2}\right]M_{U,c}R_c\left(D_c\right)-M_{S,c}S_c\leq 0 \quad (33)$$

For the case of no model uncertainty formulation, the distribution of the resistance may be updated. A likelihood of the resistance $L_{R_c(D_c)}\left(r_c\left(d_c\right)\right)$ may be obtained and used for updating the distribution of $R_c\left(D_c\right)$, see Equ. (34). Alternatively, the distribution parameters of $R_c\left(D_c\right)$ may be updated.

$$f^{''}_{R_c(D_c)}\left(r_c\left(d_c\right)\right)=f^{'}_{R_c(D_c)}\left(r_c\left(d_c\right)\right)\cdot L_{R_c(D_c)}\left(r_c\left(d_c\right)\right)\cdot\frac{1}{c_n} \quad (34)$$

If the resistance is to be updated in the context of a pre-posterior decision analysis, the resistance with non-destructive or destructive testing information should be forecasted based on the prior distribution of the resistance. For posterior updating to be utilised in a posterior decision analysis, solely the distribution of the likelihood of the observation needs to be utilised.

4.5 EQUALITY INFORMATION MODELLING

Monitoring information are defined with an equation in contrast to limit state functions, which constitute an inequality. Measurement information can thus not directly be utilised to adapt the system state probabilities (Schall, Gollwitzer et al. (1989)). However, a limit state function representing the measurement information may be derived with the uncertain measurement M_U and the prior distribution of the measurement value $m\left(\mathbf{X}_{X_{1,S}}\right)$.

$$Z_{1,S}: \quad g_{Z_{1,S}}\left(\mathbf{X}_{Z_{1,S}}, M_U\right) = M_U - m\left(\mathbf{X}_{X_{1,S}}\right) \leq 0 \tag{35}$$

With Equ. (35), the likelihood for a measurement outcome given $\mathbf{X}_{X_{1,S}} = \mathbf{x}_{X_{1,S}}$ can be formulated. It can also been shown that a limit state function to solve $P\left(X_{1,c}(t) \cap \mathbf{Z}_i(t_{i,1}) \cap \ldots \cap \mathbf{Z}_i(t_{i,n})\right)$ directly can be derived (see Straub and Papaioannou (2015) and Schneider, Thöns et al. (2017)).

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